

## The Discriminant

- **Concept:** Using the discriminant to determine the nature of the solutions of a quadratic equation

**Remember:** The discriminant  $b^2 - 4ac$  tells you how many solutions the quadratic equation has without actually solving the equation.

**Example:** Use the discriminant to determine the nature of the solutions of  $16x^2 + 8x + 1$

**Step 1** Put the equation in the form  $ax^2 + bx + c = 0$ . Then determine the values of  $a$ ,  $b$ , and  $c$ . In this case  $a = 16$ ,  $b = 8$ , and  $c = 1$ .

**Step 2** Write the discriminant  $b^2 - 4ac$  and substitute  $a = 16$ ,  $b = 8$ , and  $c = 1$ :  
 $(8)^2 - (16)(1)$

**Step 3** Simplify:  $64 - 16 = 48$

**Step 4** Use the chart in your textbook to determine the nature of the solutions. Since  $b^2 - 4ac > 0$  is not a perfect square, there are two real irrational unequal numbers.

Use the discriminant to determine the nature of the solutions of each quadratic equation.

1.  $x^2 - 6x + 9 = 0$  \_\_\_\_\_ 2.  $x^2 - 3 = 0$  \_\_\_\_\_

3.  $4x^2 + 3x - 1 = 0$  \_\_\_\_\_

- **Concept:** Determining the relationship between the nature of the solutions and the graph of a quadratic function

**Remember:** The discriminant tells us how many times the quadratic function crosses the  $x$ -axis without graphing the function.

**Example:** Describe the relation of the graph  $y = x^2 - x - 2$  to the  $x$ -axis.

**Step 1** Determine the values of  $a$ ,  $b$ , and  $c$ . In this case  $a = 1$ ,  $b = -1$ , and  $c = -2$ .

**Step 2** Write the discriminant  $b^2 - 4ac$  and substitute  $a = 1$ ,  $b = -1$ , and  $c = -2$ :  
 $(-1)^2 - 4(1)(-2)$ .

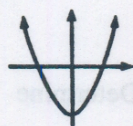
**Step 3** Simplify:  $1 + 8 = 9$

**Step 4** Use the following chart:

two  $x$ -intercepts

$b^2 - 4ac > 0$

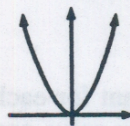
two  $x$ -intercepts



graph tangent to

$b^2 - 4ac = 0$

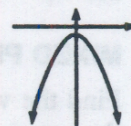
one  $x$ -intercept,  
graph tangent to  
 $x$ -axis



no  $x$ -intercepts

$b^2 - 4ac < 0$

no  $x$ -intercepts



since  $9 > 0$ , the graph crosses the  $x$ -axis in two different points.

Describe the relation of the graph of each function to the  $x$ -axis.

4.  $y = x^2 - 9$  \_\_\_\_\_

5.  $y = x^2 - 6x + 9$  \_\_\_\_\_

6.  $y = -x^2 - 2$  \_\_\_\_\_